

101 學年度第一學期第一次定期考高三數學乙試題

一、多重選擇題

1. AE

(B) 應修正為 $Var(Y) = a^2 Var(X)$ (C) 應修正為 $\sigma(Y) = |a|\sigma(X)$

(D) 應修正為 $E(X^2) = [E(X)]^2 + Var(X)$

2. BD

一顆骰子點數依除以 3 的餘數，分類為三種：

$\{3k+1 | k=0, 1\}$ 、 $\{3k+2 | k=0, 1\}$ 、 $\{3k | k=1, 2\}$

$X = 0$ ，出現之兩數可能為 $\{3k\} + \{3k\}$ 或 $\{3k+1\} + \{3k+2\}$

X	0	1	2
$P(x=k)$	$\frac{C_1^2 \cdot C_1^2 + 2 \cdot C_1^2 \cdot C_1^2}{36} = \frac{1}{3}$	$\frac{2 \cdot C_1^2 \cdot C_1^2 + C_1^2 \cdot C_1^2}{36} = \frac{1}{3}$	$\frac{2 \cdot C_1^2 \cdot C_1^2 + C_1^2 \cdot C_1^2}{36} = \frac{1}{3}$

$X = 1$ ，出現之兩數可能為 $\{3k\} + \{3k+1\}$ 或 $\{3k+2\} + \{3k+2\}$

$X = 2$ ，出現之兩數可能為 $\{3k\} + \{3k+2\}$ 或 $\{3k+1\} + \{3k+1\}$

(A) $P(X=0) = \frac{1}{3}$

(B) $Var(X) = E(X^2) - [E(X)]^2 = (1^2 \cdot \frac{1}{3} + 2^2 \cdot \frac{1}{3}) - (1 \cdot \frac{1}{3} + 2 \cdot \frac{1}{3})^2 = \frac{5}{3} - 1^2 = \frac{2}{3}$

(C) $P(X \leq 1) = P(X=0) + P(X=1) = \frac{2}{3}$

(D) $P(X=2) = \frac{1}{3}$

(E) $P(X=1) = P(X=2) = \frac{1}{3}$

3. ABE

$$P(A) = \frac{C_1^9}{C_2^{10}} = \frac{9}{45} = \frac{1}{5}, \quad P(B) = \frac{C_2^5 + C_2^5}{C_2^{10}} = \frac{20}{45} = \frac{4}{9},$$

$$P(C) = 1 - \frac{C_2^5}{C_2^{10}} = 1 - \frac{10}{45} = \frac{35}{45} = \frac{7}{9}$$

$$P(A \cap B) = \frac{C_1^4}{C_2^{10}} = \frac{4}{45}, \quad P(B \cap C) = \frac{C_2^5}{C_2^{10}} = \frac{10}{45} = \frac{2}{9}, \quad P(A \cap C) = \frac{C_1^5}{C_2^{10}} = \frac{5}{45} = \frac{1}{9}$$

(A) $\frac{1}{5} < \frac{4}{9} < \frac{7}{9} \Rightarrow P(A) < P(B) < P(C)$ (B) $P(A \cap B) = \frac{4}{45} = P(A) \cdot P(B)$ ，所

以 A 、 B 為獨立事件

(C) $P(B \cap C) = \frac{2}{9} \neq P(B) \cdot P(C)$ ，所以 B 、 C 不為獨立事件

(D) $P(A \cap C) = \frac{1}{9} \neq P(A) \cdot P(C)$ ，所以 A 、 C 不為獨立事件

(E) 因 $A \cap C$ 表兩點數為 1 奇 1 偶的事件， B 為點數和為偶數的事件，故

$$(A \cap C) \cap B = \phi, \text{ 故 } P(A \cap B \cap C) = 0$$

4. DE

(A) $(\frac{2}{3})^5 = \frac{32}{243}$ (B) $(\frac{1}{3})^5 = \frac{1}{243}$ (C) $(\frac{2}{3})^4 \cdot \frac{1}{3} = \frac{16}{243}$ (D)(E)

$$(\frac{2}{3})^3 \cdot (\frac{1}{3})^2 = \frac{8}{243}$$

5. ACDE

因甲只需再勝一場即可獲勝，故接下來的比賽，只有可能為(甲勝)、(乙勝，甲勝)、(乙勝，乙勝)

(A) 所以最多只需再進行兩場比賽。

(B) 乙獲得最後勝利的機率為 $(\frac{1}{2})^2 = \frac{1}{4}$

(C) 甲獲得最後勝利的機率為 $1 - (\frac{1}{2})^2 = \frac{3}{4}$

(E) 故甲獲得獎金的期望值為 $10000 \times \frac{3}{4} = 7500$

二、填充題

1. (1) $\frac{11}{12}$ (2) $\frac{1}{3}$

$$(1) P(A \cup B) = P(B) + P(A \cap B') = \frac{2}{3} + \frac{1}{4} = \frac{11}{12}$$

$$(2) P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{\frac{1}{3} \times (1 - \frac{2}{3})}{1 - \frac{2}{3}} = \frac{1}{3}$$

2. (1) $\frac{13}{6}$ (2) $\frac{23}{36}$

$$(1) E(X) = 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{5}{12} = \frac{13}{6}$$

$$(2) Var(X) = E(X^2) - [E(X)]^2 = (1^2 \cdot \frac{1}{4} + 2^2 \cdot \frac{1}{3} + 3^2 \cdot \frac{5}{12}) - (\frac{13}{6})^2 = \frac{16}{3} - \frac{169}{36} = \frac{23}{36}$$

3. 10

$$E(2X - 3) = E(X) \Rightarrow 2E(X) - 3 = E(X) \Rightarrow E(X) = 3$$

$$Var(3X) = 9Var(X) = 9 \Rightarrow Var(X) = 1$$

$$Var(X) = E(X^2) - [E(X)]^2 \Rightarrow E(X^2) = 10$$

4. 10

$$\text{所求期望值為 } 50 \times \frac{1}{8} + 10 \times \frac{3}{8} = \frac{80}{8} = 10$$

5. $\frac{13}{10}$

隨機變數 X 之機率分布為：

k	1	2	3	4
$P(x=k)$	$\frac{9}{12} = \frac{3}{4}$	$\frac{3 \cdot 9}{12 \cdot 11} = \frac{9}{44}$	$\frac{3 \cdot 2 \cdot 9}{12 \cdot 11 \cdot 10} = \frac{9}{220}$	$\frac{3 \cdot 2 \cdot 1 \cdot 9}{12 \cdot 11 \cdot 10 \cdot 9} = \frac{1}{220}$

$$\text{所以 } E(X) = 1 \cdot \frac{3}{4} + 2 \cdot \frac{9}{44} + 3 \cdot \frac{9}{220} + 4 \cdot \frac{1}{220} = \frac{13}{10}$$

6. 13000