

## 二、因式分解

### 2-1

- 【類題練習 1】** (1) 商式 =  $2x - 5$  ; 餘式 = 20                      (2) 商式 =  $-3x + 1$  ; 餘式 = 2  
(3) 商式 =  $x^3 + x^2 + x + 1$  ; 餘式 = 0                      (4) 商式 =  $2x - 5$  ; 餘式 = 25

**【家庭作業】**

1. ① 商式 =  $3x + 2$  ; 餘式 = 0  
② 商式 =  $\frac{7}{2}x + \frac{1}{4}$  ; 餘式 =  $-\frac{15}{4}$   
③ 商式 =  $x^2 + x + 1$  ; 餘式 = 2  
④ 商式 =  $x^2 + 5x + 27$  ; 餘式 = 134  
⑤ 商式 =  $x^2 - x + 5$  ; 餘式 =  $-18x + 14$   
⑥ 商式 =  $x^2 + 1$  ; 餘式 = 2

2.  $a = 1, b = 2$                       3.  $2x^3 - 5x^2 + 4x + 2$   
4.  $-6$                                       5.  $x - 1$

6. 設  $B \div (x - 2)$  的商式為  $Q$ ,  $B = (x - 2)Q + (-2)$

$$\begin{aligned} \because A &= (2x + 1)B + 3 && \text{代入} \\ &= (2x + 1)[(x - 2)Q - 2] + 3 \\ &= (2x + 1)(x - 2)Q - 2(2x + 1) + 3 \\ &= (2x + 1)(x - 2)Q - 4x + 1 \end{aligned}$$

所以餘式為  $-4x + 1$ .

7. 因為被除式 =  $(x - 1) \times$  商式 + 餘式, 其中餘式必為常數,  
所以設  $(x^2 - 1)^{10} + x^2 + x - 1 = (x - 1)B + C$ ,  $C$  為餘式.  
以  $x = 1$  代入上式可得,  $1 = C$ , 所以餘式為 1.

### 2-2

- 【類題練習 1】** (1)  $2x(2x + 3)$                       (2)  $(a + b)(7a + 7b - 3)$                       (3)  $(x - y)^2(1 - x + y)$

- 【類題練習 2】** (1)  $(x - 1)(x^2 + 1)$                       (2)  $(2y - 3)(x + 2)$                       (3)  $(5ax - 2)(x + 1)$   
(4)  $(a - bc)(ac + b)$

**【家庭作業】**

1. ①  $x(2 - a)$                       ②  $3ab(a + 2b)$                       ③  $x^2$   
④  $-3(a - 2)(b + 3)$                       ⑤  $-(a - 3)^2$                       ⑥  $(2b - 1)(a + 3)$

2. ①  $(x - 2)^2 + 2x - 4 = (x - 2)^2 + 2(x - 2)$   
 $= (x - 2)(x - 2 + 2) = x(x - 2)$

$$\begin{aligned}
 \textcircled{2} \quad (x-2)^3 + (2-x)(x^2 - 4x + 1) &= (x-2)^3 - (x-2)(x^2 - 4x + 1) \\
 &= (x-2)[(x-2)^2 - (x^2 - 4x + 1)] \\
 &= (x-2)[(x^2 - 4x + 4 - (x^2 - 4x + 1))] \\
 &= 3(x-2)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \quad (ax-bx)^2 - (b-a)^3 x &= (a-b)^2 x^2 + (a-b)^3 x \\
 &= x(a-b)^2 [x + (a-b)] \\
 &= x(a-b)^2 (x+a-b)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{4} \quad x^3 + 2x^2 + 2x + 1 &= x^3 + x^2 + x + x^2 + x + 1 \\
 &= x(x^2 + x + 1) + (x^2 + x + 1) \\
 &= (x+1)(x^2 + x + 1)
 \end{aligned}$$

### 2-3

**【類題練習 1】** (1)  $(5x+17)(x-3)$  (2)  $(19-x)(20+x)$

**【類題練習 2】** (1)  $(4x+3)(x-2)/2$  (2)  $(2x-5)(3x+1)/5$

**【家庭作業】**

1. ①  $(x+3)(x+11)$  ②  $5(x+1)(x-2)$   
 ③  $(2x-5)(x+4)/2$  ④  $(9x+1)(x-4)$   
 ⑤  $7(a-5b)(a+3b)$  ⑥  $(2x-2y-5)(x-y+1)$   
 ⑦  $(x-p)(x+q)$  ⑧  $(ax+b)(x-1)$

2. ①  $4x^4 - 13x^2 - 12 = (4x^2 + 3)(x^2 - 4) = (4x^2 + 3)(x+2)(x-2)$

$$\begin{aligned}
 \textcircled{2} \quad (a+b)(a+b-4) - 12 &= (a+b)[(a+b)-4] - 12 \\
 &= (a+b)^2 - 4(a+b) - 12 \\
 &= (a+b-6)(a+b+2)
 \end{aligned}$$

$$\begin{aligned}
 \textcircled{3} \quad (x-4y)(x+4y) + 6xy &= x^2 - 16y^2 + 6xy \\
 &= x^2 + 6xy - 16y^2 \\
 &= (x+8y)(x-2y)
 \end{aligned}$$

$$\textcircled{4} \quad x^2 - (a + \frac{1}{a})x + 1 = x^2 - ax - \frac{1}{a}x + a \cdot \frac{1}{a} = (x-a)(x - \frac{1}{a})$$

$$\begin{aligned}
 \textcircled{5} \quad (x^2 - x + 1)^2 - 3(x^2 - x) - 7 \\
 &= (x^2 - x + 1)^2 - 3(x^2 - x + 1 - 1) - 7 \\
 &= (x^2 - x + 1)^2 - 3(x^2 - x + 1) - 4 \\
 &= (x^2 - x + 1 - 4)(x^2 - x + 1 + 1) \\
 &= (x^2 - x - 3)(x^2 - x + 2)
 \end{aligned}$$

$$\begin{aligned}
& \textcircled{6} (x^2 + 3x + 5)(x^2 + 3x + 1) + 3 \\
&= (x^2 + 3x + 1 + 4)(x^2 + 3x + 1) + 3 \\
&= (x^2 + 3x + 1)^2 + 4(x^2 + 3x + 1) + 3 \\
&= (x^2 + 3x + 1 + 3)(x^2 + 3x + 1 + 1) \\
&= (x + 1)(x + 2)(x^2 + 3x + 4)
\end{aligned}$$

## 2-4

**【類題練習 1】** (1)  $(a+5)^2$  (2)  $(4x-5y)^2$  (3)  $4(2x-3y)^2$  (4)  $(a-b+c)^2$

**【類題練習 2】** (1)  $(a+1)^2(a-1)^2$  (2)  $(2x+1)^2$  (3)  $(a+b-1)(a-b+1)$   
(4)  $(x^2 + y^2)(x+y)(x-y)$

**【類題練習 3】** (1)  $(x-1)^3$  (2)  $(2x-y)^3$  (3)  $(3+x)^3$  (4)  $(3x+2y)^3$

**【類題練習 4】** (1)  $(x + \frac{1}{3})(x^2 - \frac{1}{3}x + \frac{1}{9})$  (2)  $(2a-5b)(4a^2 + 10ab + 25b^2)$

(3)  $(x-1)(x^2 + 2x + 2)$

(4)  $(a+2b)(a^2 - 2ab + 4b^2)(a-2b)(a^2 + 2ab + 4b^2)$

**【家庭作業】** 1. ①  $(x+7)^2$  ②  $3(x-2)^2$  ③  $(x+2a-2b)^2$

④  $2(x+3)(x-3)$  ⑤  $-(7+2a)(5+2a)/4$

⑥  $(a+2b)^2(a-2b)^2$  ⑦  $2(x+2y)(x^2 - 2xy + 4y^2)$

⑧  $(5x-2)(25x^2 + 10x + 4)$

2. ①  $x^2 - y^2 + 6yz - 9z^2 = x^2 - (y^2 - 6yz + 9z^2)$

$$= x^2 - (y-3z)^2$$

$$= (x+y-3z)(x-y+3z)$$

②  $(1-ab)^2 - (a-b)^2 = [(1-ab) + (a-b)][(1-ab) - (a-b)]$

$$= (1+a-b-ab)(1-a+b-ab)$$

$$= [(1+a)-b(1+a)][(1-a)+b(1-a)]$$

$$= (1+a)(1-b)(1-a)(1+b)$$

③  $(a^2-1)(b^2-1) - 4ab = a^2b^2 - b^2 - a^2 + 1 - 4ab$

$$= (a^2b^2 - 2ab + 1) - (a^2 + 2ab + b^2)$$

$$= (ab-1)^2 - (a+b)^2$$

$$= (ab-1-a-b)(ab-1+a+b)$$

$$\textcircled{4} \quad \frac{1}{4}a^2 - \frac{2}{3}a + \frac{4}{9} = \frac{1}{36}(9a^2 - 24a + 16) = \frac{1}{36}(3a-4)^2$$

$$\begin{aligned} \textcircled{5} \quad x^3 + x^2 - 36 &= x^3 - 27 + x^2 - 9 \\ &= (x-3)(x^2 + 3x + 9) + (x-3)(x+3) \\ &= (x-3)(x^2 + 3x + 9 + x + 3) \\ &= (x-3)(x^2 + 4x + 12) \end{aligned}$$

$$\begin{aligned} \textcircled{6} \quad x^4 - x^3 + 4x^2 - 3x + 3 &= x^4 - x^3 + x^2 + 3x^2 - 3x + 3 \\ &= x^2(x^2 - x + 1) + 3(x^2 - x + 1) \\ &= (x^2 - x + 1)(x^2 + 3) \end{aligned}$$

$$3. \quad \textcircled{1} \quad a^2 + b^2 = (a+b)^2 - 2ab = 3^2 - 2 \times 2 = 5$$

$$\textcircled{2} \quad 4a^2 - ab + 4b^2 = 4(a^2 + b^2) - ab = 4 \times 5 - 2 = 18$$

$$\begin{aligned} \textcircled{3} \quad a^3 + b^3 &= (a+b)(a^2 - ab + b^2) \\ &= (a+b)(a^2 + b^2 - ab) = 3 \times (5 - 2) = 9 \end{aligned}$$

## 2-5

**【類題練習 1】** (1)  $(x+3)(x-1)$  (2)  $(5a-2)(a-2)$   
 (3)  $(a^2 + ab + b^2)(a^2 - ab + b^2)$  (4)  $(3x^2 + x + 2)(3x^2 - x + 2)$

**【類題練習 2】** (1)  $(x+4+\sqrt{7})(x+4-\sqrt{7})$  (2)  $(x+y)(x^2 - xy + y^2)$   
 (3)  $(x^2 + 4x + 8)(x^2 - 4x + 8)$

**【家庭作業】** 1.  $\textcircled{1} (x+2)(x+4)$   $\textcircled{2} (5a^2 + 2a + 1)(5a^2 - 2a + 1)$   
 $\textcircled{3} (x^2 + 6x + 18)(x^2 - 6x + 18)$   $\textcircled{4} (a^2 - b)(a^2 - 3b)$

$$\begin{aligned} 2. \quad \textcircled{1} \quad x^2 + 10x + 23 &= x^2 + 2 \cdot x \cdot 5 + 5^2 - 5^2 + 23 \\ &= (x+5)^2 - 2 \\ &= (x+5)^2 - (\sqrt{2})^2 \\ &= (x+5+\sqrt{2})(x+5-\sqrt{2}) \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad a^2 - b^2 - c^2 + 2bc + a + b - c \\ &= a^2 - (b^2 - 2bc + c^2) + (a + b - c) \\ &= a^2 - (b-c)^2 + (a + b - c) \\ &= (a+b-c)(a-b+c) + (a+b-c) \\ &= (a+b-c)(a-b+c+1) \end{aligned}$$

$$\begin{aligned}
③ \quad & 8a^3 - 1 \\
&= (2a)^3 - 3(2a)^2 \cdot 1 + 3(2a) \cdot 1^2 - 1^3 + 3(2a)^2 \cdot 1 - 3(2a) \cdot 1^2 \\
&= (2a - 1)^3 + 6(2a - 1) \\
&= (2a - 1)[(2a - 1)^2 + 6] \\
&= (2a - 1)(4a^2 + 2a + 1)
\end{aligned}$$

$$\begin{aligned}
④ \quad & 27x^3 + 8y^3 \\
&= (3x)^3 + 3 \cdot (3x)^2 \cdot 2y + 3 \cdot 3x \cdot (2y)^2 + (2y)^3 - 54x^2y - 36xy^2 \\
&= (3x + 2y)^3 - 18xy(3x + 2y) \\
&= (3x + 2y)[(3x + 2y)^2 - 18xy] \\
&= (3x + 2y)(9x^2 - 6xy + 4y^2)
\end{aligned}$$

$$3. \quad ① \quad \because a - c = (a - b) + (b - c) = 5 + 3 = 8$$

$$\begin{aligned}
&\therefore a^2 + b^2 + c^2 - ab - bc - ca \\
&= (2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca) / 2 \\
&= [(a^2 - 2ab + b^2) + (b^2 - 2bc + c^2) + (c^2 - 2ca + a^2)] / 2 \\
&= [(a - b)^2 + (b - c)^2 + (c - a)^2] / 2 \\
&= [5^2 + 3^2 + 8^2] / 2 = 49
\end{aligned}$$

$$\begin{aligned}
② \quad & (x + a)^2 + (y + b)^2 + (z + c)^2 \\
&= x^2 + y^2 + z^2 - 6x - 4y + 10z + 38 \\
&= (x^2 - 6x + 9) + (y^2 - 4y + 4) + (z^2 + 10z + 25) \\
&= (x - 3)^2 + (y - 2)^2 + (z + 5)^2
\end{aligned}$$

所以,  $a = -3, b = -2, c = 5$ .

$$4. \quad ① \quad a^2 - 2ab + b^2 - 2a + 2b + 1$$

$$\begin{aligned}
&= (a - b)^2 - 2(a - b) + 1 \\
&= (a - b - 1)^2
\end{aligned}$$

$$② \quad a^2 - 4b = b^2 + 4a \Rightarrow a^2 - b^2 - 4a - 4b = 0$$

$$\Rightarrow (a - b)(a + b) - 4(a + b) = 0$$

$$\Rightarrow (a + b)(a - b - 4) = 0$$

因爲  $a + b > 0$ , 所以  $a - b - 4 = 0 \Rightarrow a - b = 4$

$$③ \quad \text{承}②, \quad a^2 - 2ab + b^2 - 2a + 2b + 1$$

$$= (a - b - 1)^2 = (4 - 1)^2 = 9$$